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Power output estimation and experimental validation for piezoelectric energy harvesting systems

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Abstract Many modern devices especially for ubiquitous computing or wireless sensor networks need a long life energy source. Batteries or accumulators are often an insufficient solution. Low frequency vibrations can be found in the most technical facilities or even in the human movements. Even while these vibrations are neither wanted nor used in the most times, they enable us to generate electrical energy. Piezoelectric flexural transducers are a promising choice for utilizing the vibrations for energy harvesting. There are two major influences on the amount of generated energy. First there is the frequency behavior of the piezoelectric transducers, for optimal power output the transducer should be driven in resonance. Second, the energy output is highly dependent on the electrical load of the connected application. Both circumstances, working frequency and electrical load, typically are boundary conditions for the development of the generator. Therefore, it is necessary to handpick the type of piezoelectric elements. To meet the requirements of development engineers, a model based design method for energy harvesting systems is needed. As a first step towards such a method, this work proposes a model for the estimations of the power output of piezoelectric flexural transducers. For the validation of this model an experiment is described in detail. The results of the model and the experiments are compared.

Keywords Energy harvesting · Piezoelectric elements · Autonomous systems · Energy supply

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1 Introduction

Energy Harvesting methods are able to provide electric energy for autonomous applications using ambient energy. Examples for autonomous systems are wireless sensors or consumer electronics. The properly best argument for the use of energy harvesting devices is the long lifetime of the energy source and the so enlarged maintenance intervals. The requirements for an autonomous energy source are formal pretty simple: First, there must be enough energy in the environment. Second we need a transducer that is able to harvest this energy. Often, the surrounding energy is kinetic energy. To harvest such energy, piezoelectric transducers seems to be a suitable solution.

The big challenge during the design process of an energy harvesting system is to find the best fitting transducer. Before beginning the design it is essential to gain a good knowledge about the harvesting process. Our approach is starting with a model with the low number of parameters to explain the general behavior of the piezoelectric generator.

In this contribution we introduce a suitable one degree of freedom model for piezoelectric generators and its parameter identification. For validation of the model measurements are performed and analyzed.

2 Modeling and parameter identification

A good approximation of the piezoelectric generator is essential for their design process. Finding a suitable mathematical description is the main objective in this section.

The model approach is based on the electromechanical analogies and will provide a modal one degree of freedom model. Here we use the analogy where $F \propto U$ and $x \propto q$. Therefore the mechanical stiffness c can be represented by a capacitor $\frac{1}{C}$ in the electrical model. Damping d can be described by a resistance R and the mass m by the inductance L.

The schematic equivalent system representation of the proposed model is shown in Fig. 1. The System can be represented using mechanical symbols (a) as well as electrical symbols (b), both schematics leads to identical mathematical equations. In Fig. 1(a) the lower part represents the modal mechanical behavior including the excitation at the base point of the piezoelectric generator. The upper part depicts the electrical behavior of the piezoelectric element and the electrical load at the generator. The coupling between the mechanical and electrical subsystem is modeled as lever, where the length α has the dimension $[\frac{N}{V}]$.

This model is only valid, when the base velocity $\dot{y} = v_{base}$ is small against the tip velocity $\dot{x} = v_{tip}$. Here v_{base} is defined as excitation velocity of the base and v_{tip} is defined as velocity of the transducers tip in relation to the velocity of the base. Due to the modal approach, the model should be used in a small range around the identified mode. In general, the range of validity can be expanded by adding further modes to the model. For operation in or close to one resonance frequency the depicted model—using only one mode—is sufficient. The mechanical representation, using the parameters m for the modal mass, the modal stiffness c_m, the modal damping d_m, the piezoelectric capacity $c_p = \frac{1}{C_p}$, the resistive load of the driven application R_{load}, and the transmission factor α leads to the derivation of the equations of motion as follow:

$$c_m(x(t)-y(t)) + d_m(\dot{x}(t)-\dot{y}(t)) + m\ddot{x}(t) = -\alpha F(t) \tag{1}$$

$$c_p(\alpha x(t) - q(t)) = F(t) \tag{2}$$

$$R_{load}\dot{q}(t) = F(t) \tag{3}$$

The transfer functions for the generated voltage and for the power at the load can be written in the Laplace space:

$$U(s) = \frac{(c_m + d_m s)\alpha R_{Load}}{sR_{Load}\,\alpha^2 + (c_m + s(d_m + m\,s))\left(1 + s\,C_p R_{Load}\right)} v_{base} \tag{4}$$

$$P(s) = U(s)\left(\frac{U(s)}{R_{Load}}\right)^* \tag{5}$$

Using the transfer functions, it is possible to determine the output power with varying frequencies and resistances. Therefore, the parameters of the equivalent system model have to be identified.

Concerning the similarities between the proposed generator model and the commonly used actuator model (cp. Figs. 1 and 2) it is possible to use a similar identification technique. For identification of all parameters beside α the measurement of the electrical admittance ($Y_{el} = \frac{i}{u} = \frac{\dot{q}}{u}$) is sufficient [1, 2]. The identification of the parameters can be done utilizing the frequency response locus shown in Fig. 2(b) and the following equations:

$$R_m = \frac{1}{\max(\Re(Y_{el}))} \tag{6}$$

$$C_p = \frac{1}{c_p} = \frac{\max(\Im(Y_{el})) - \min(\Im(Y_{el}))}{2\omega_r} \tag{7}$$

$$L_m = \frac{R_m}{\omega_1 - \omega_2} = \frac{R_m}{\omega(\max(\Im(Y_{el}))) - \omega(\min(\Im(Y_{el})))} \tag{8}$$

$$C_m = L_m \omega_r^2 \tag{9}$$

The missing parameter α can be determined by an additional measurement of the mechanical admittance ($Y_{mech} = \frac{v_{tip}}{u}$). The parameter is identified using the ratio between the maxima of both admittances. The

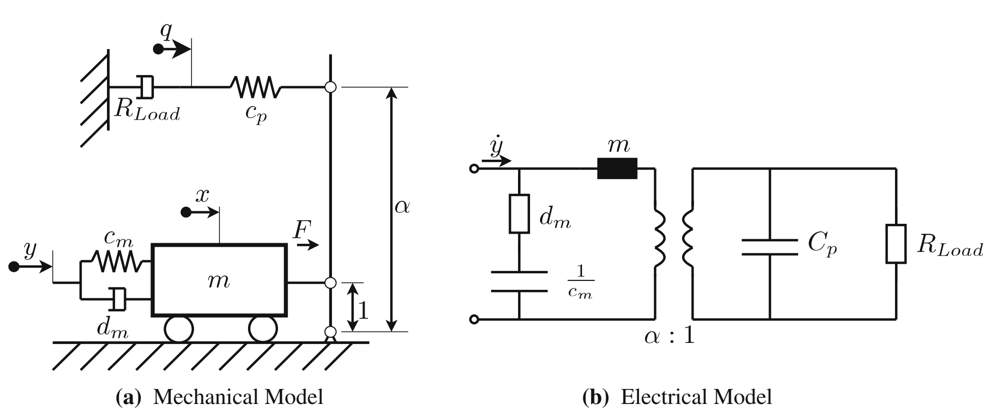
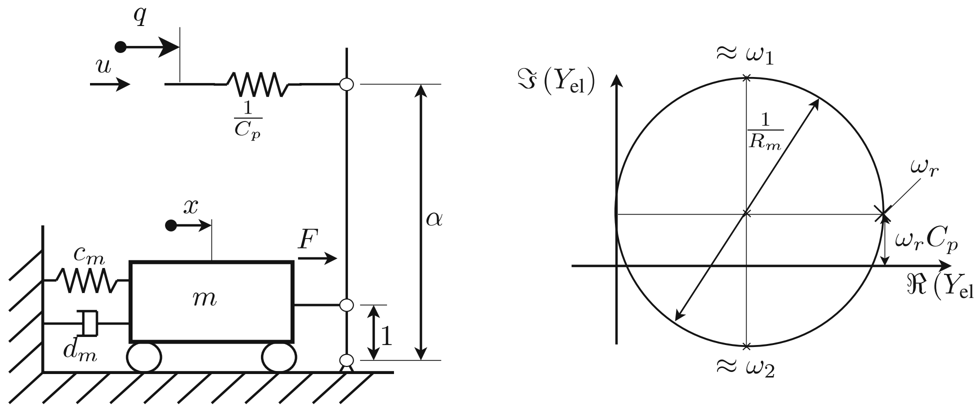


Fig. 1 Equivalent models for a base excited piezoelectric generator

Fig. 2 Parameter identification



(a) Mechanical equivalent model for a piezoelectric actuator

(b) Ideal frequency response locus

knowledge of α allows the calculation of all model parameters:

$$\alpha = \frac{\max(\Re(Y_{el}))}{\max(\Re(Y_{mech}))} = \frac{1}{R_m \max(\Re(Y_{mech}))} \tag{10}$$

$$d_m = \alpha^2 R_m \tag{11}$$

$$m = \alpha^2 L_m \tag{12}$$

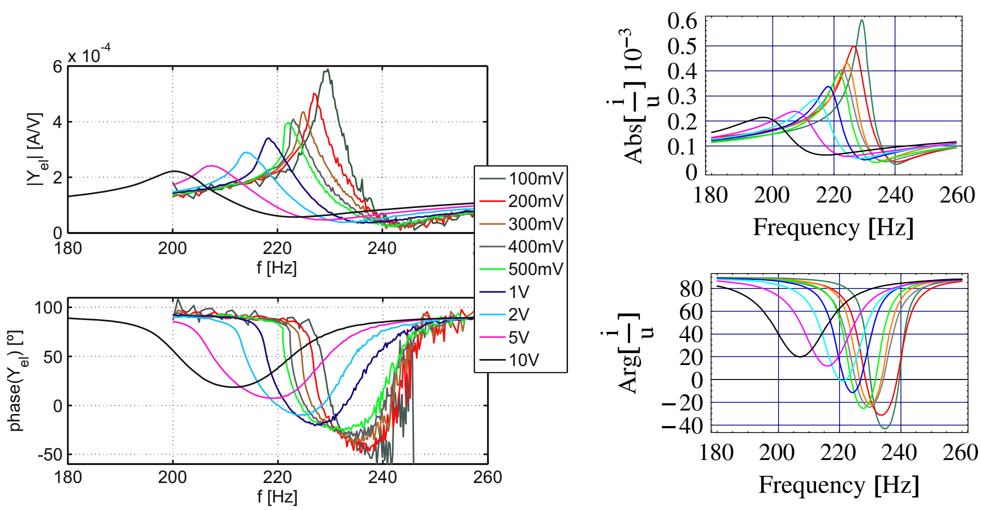
$$c_m = \frac{\alpha^2}{C_m} \tag{13}$$

$$Q_m = \frac{1}{R_m}\sqrt{\frac{L_m}{C_m}} \tag{14}$$

Parameter identifications for further model analysis are performed (piezoelectric element: Argillon Type "Trimorph—carbonfiber for SITEX-Modules"). Due to the strong dependence of the parameters to the excitation voltage, parameters are determined for multiple excitation levels. The electrical response is depicted in Fig. 3(a) and the identified systems are shown in Fig. 3(b). The resulting parameters can also be seen in Table 1. The damping parameter is the one with the strongest dependence to the excitation level. The identification measurements points out that it is necessary to use separated parameter-sets for each excitation level.

Using Eq. 5 for power output, the optimal working point regarding load and frequency can be determined. Varying both, frequency and load resistance, gives the plots in Fig. 4 (calculated with constant base velocity). The influence of the damping is easy to see: The lower damped system has two good operation conditions: One optimum at the resonance frequency of the unloaded system and one at the anti-resonance frequency of the unloaded system. Unlike this, the

Fig. 3 Frequency responses Y_{el} of piezoelectric element with different excitation amplitudes



(a) Measurement

(b) Identified Models

Table 1 Identified parameter.

Excitation-level [V]	m [10^{-6} kg]	d_m [$10^{-3}\,\frac{\text{Ns}}{\text{m}}$]	c_m [$\frac{\text{N}}{\text{m}}$]	α [$10^{-3}\,\frac{\text{N}}{\text{V}}$]	Cp [10^{-9} F]	f_r [Hz]	Q_m
0.20	312.2	13.730	637.5	2.528	89.3	227	32.5
0.30	399.7	16.325	801.0	2.528	91.7	225	34.7
0.50	416.7	17.018	815.6	2.473	79.7	223	34.3
1.0	412.9	20.106	785.5	2.440	86.1	220	28.3
5.0	338.8	30.332	592.0	2.380	84.7	210	14.8
10.0	323.7	35.623	517.0	2.385	87.5	201	11.5

strongest damped system has only one optimal operation condition at a frequency between resonance and anti-resonance. Further, a relationship between the phase diagram (lower part of Fig. 3(a) and the number of maxima can be found and explained: The weaker damped system (500 mV excitation level) has two optimal operation conditions. Both are at the frequency of the phase 0° crossing where only effective power is generated. With increasing damping, the minimum phase is raising and as long there is still a Phase 0° crossing there are two maxima in the system. If there is a tangential touching of the phase 0° line or a higher minimum phase only one optimal operation point occurs, close to the frequency of minimum phase where the highest effective power is generated. Combining that with the figure of merit $M = \frac{1}{\omega_r\, R_m\, C_p}$ [1], which is greater than 2 if the system has a phase 0° crossing, we get two maxima for $M > 2$ and one maximum for $M \leq 2$. The calculated output power can be used for the selection of suitable combination between piezoelectric generator and electrical load.

3 Experimental investigations

While parameter identification is done with electrical excitation, further experiments are performed with mechanical excitation. To ensure comparable mechanical conditions in the piezoelectric element during the parameter identification as well as during harvesting operation, the tip velocity at 200 Hz with the electrical excitations is noticed. For the experiments with mechanical excitation the voltage level of the shaker is adjusted to gain identical tip velocity at 200 Hz. Therefore, we assume that the behavior of the shaker is nearly constant in the interesting frequency range.

For the validation of the model, representative values for the load resistance are selected, including the calculated optimal loads. Using this selection, measurements of the generated voltage at the load over the frequency were made for different excitation levels. Those measurements were performed in three steps: First, the resistance was adjusted. Second the amplitude (v_t) was attuned to the same level than in

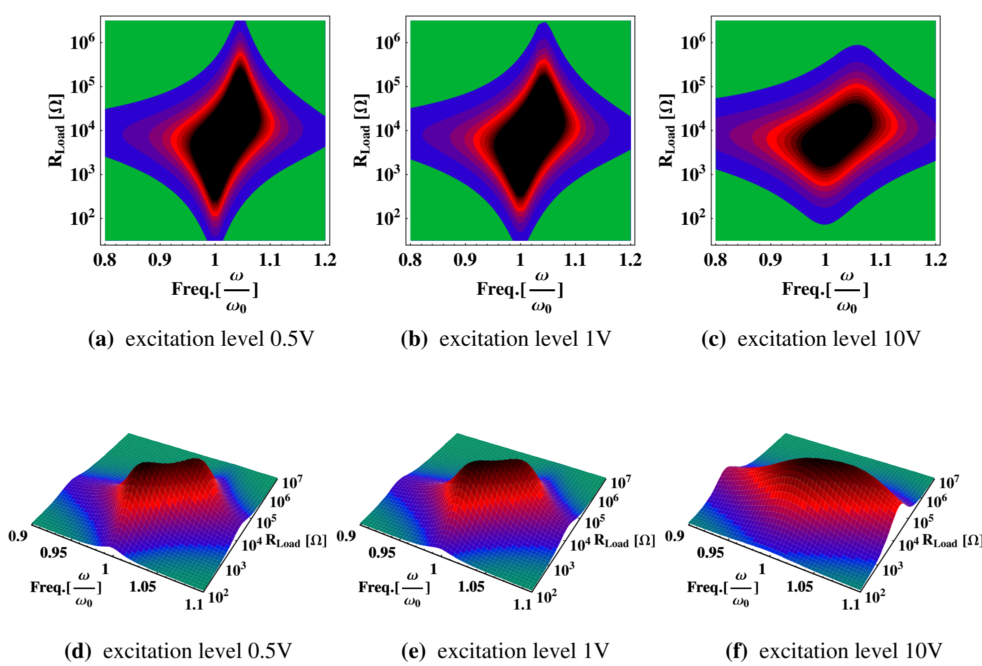
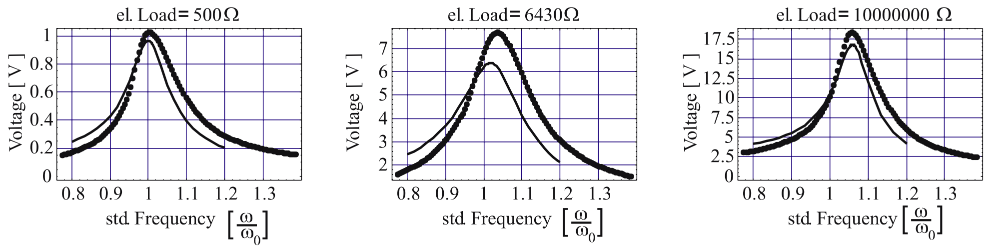
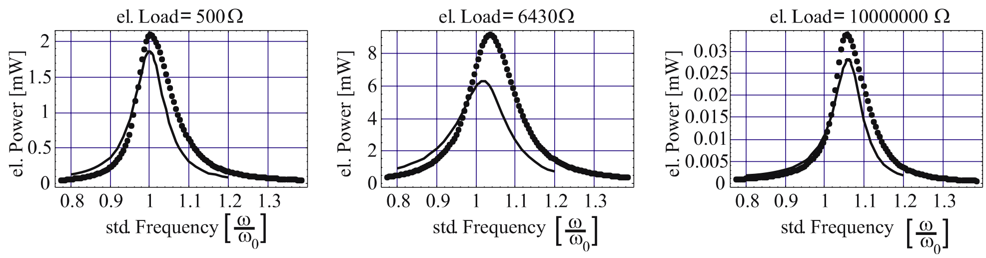


Fig. 4 Calculated power output over frequency and electrical load

Fig. 5 Results of voltage and power measurements (*dots*) and simulations (*line*) at excitation level 10 V, $v_{base} \approx 78$ mm/s



(a) generated voltage



(b) generated power

Y_{el}-measurements. Third, the generated voltage was measured with a sweep over the frequency. Results of these measurements are shown (for the 10 V excitation level which is corresponding to a v_{base} amplitude of 78 mm/s at 200 Hz) in Fig. 5(a). As expected, generated voltage is raising with increasing resistance. Further, the resonance frequency is shifting toward higher frequencies. The generated electrical power (Fig. 5(b) is calculated using Eq. 5. The gained electrical power—for the different excitation levels and different loads—ranges between a few pW to nearly 10 mW.

In both figures the voltage respective the power output estimated by the Eqs. 4 and 5 is displayed. The only missing input parameter v_{base} was measured, after adjusting v_t. As the value of v_{base} is changing with the frequency the maximum value is used as reference.

In general, the Fig. 5 shows a good matching between the measurements (dots) and the model (line). Especially in the non-optimal load cases the model delivers a suitable approximation. In case of optimal load the generated voltage is underestimated. A similar behavior has been found for other excitation levels, where a tendency of underestimation is detected in every case.

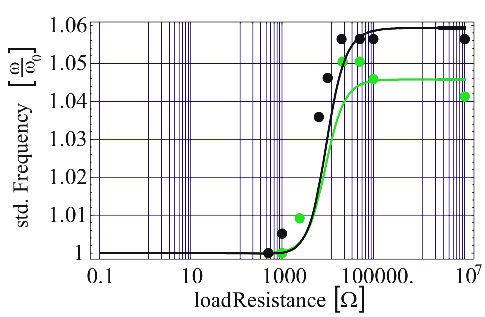
A further comparison between model and measurement is done in Fig. 6(a). It displays the change of frequency at maximum output power for changing electrical load resistance. Measurements are again dots and the model is represented by a line. The diagram points out a good matching between measurement and model.

Figure 6(a) points out the normalized power output over the load resistance at resonance frequency, this is the maximal possible power output for each electrical load (constant v_{base}). So it is a cut through the diagrams Fig. 4(d) (green line) and Fig. 4(f) (black line) always in resonance plotted versus the electrical load R_{Load}. Here a good quantitative matching is displayed. This kind of diagram is useful to choose a piezoelectric energy harvesting system for a given application.

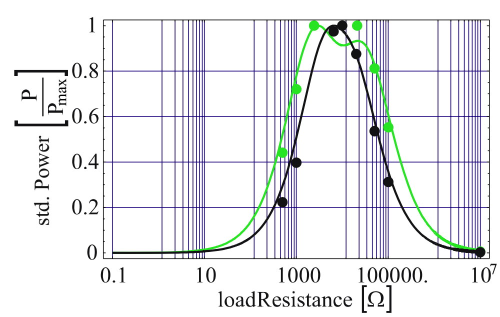
4 Conclusion and outlook

Altogether, the model matches the measurements quite acceptable. But there are still open questions: why is the fitting in the non-optimal working range better than in

Fig. 6 Comparison of measurement and model (*black*—excitation level 10 V, *green*—excitation level 0.5 V)



(a) Freq vs. load at maximal power



(b) Output power vs. load at resonance the ideal working range? And why do we have to chose each the highest v_{base} as input to get a proper matching?

Our hypothesis is that both effects are founded on the non-linear properties of the piezoelectric flexural element: In our experiments we adjusted the excitation level by comparing v_t at one frequency only. But as v_{base} is frequency dependent, v_t was not constant over the frequency. This causes changing excitation levels over the frequency and as a consequence the strain in the piezoelectric element is also changing. Even at constant v_{base}, resonance will lead to a non-constant v_t.

There are two possibilities for experiments to find the described influence of the two effects: First measurements with a constant v_{base} over the frequency, so one influence would be eliminated. For this approach we need to control velocity, due to the frequency dependency of the shaker. A second approach would be the control of the velocity v_t with the advantage that the strain in the piezoelectric element is constant over the frequency. We plan doing the measurements with constant v_t as next step.

In terms of applications a maximal output power of nearly 10mW was generated, which is enough to power wireless applications. Furthermore, the results point clearly out, that an appreciate load adaptation of the energy harvesting system is necessary. Right now our model is a useful tool for estimating the optimal electrical load. On the other way around, the model allows to check an energy harvesting system on suitability for an application. In a further step the determination of the equivalent parameter from the geometric dimensions and material parameters has to be done. With that step it will be possible to find the optimal piezoelectric flexural transducer for a given application, by calculation of theoretical optimal dimensions of the piezoelectric element for a power generation task.

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